GEOMETRIC DEFINITION OF DRUSE CRYSTAL IN PLANT CELLS

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In this study, we determined that micromorphological structures of some plant crystal have geometric structures and mathematical formulas.Plant crystals are the storage of many mineral acid salts (inorganic salts) in many plants, such as chloride, phosphate, carbonate, silicate anhydrides and sulfates, which are formed as a result of metabolism. The crystals formed take different shapes. They are named according to these shapes. One of these is called as druse crystal. In our study, it was determined that the microscopic structures of the druse crystals show a minimal surface feature, which has an important place in mathematics. Minimal surfaces are described as surfaces with zero mean curvature. Minimal surface that parametrized as x= (u,v,h (u,v)) so satisfiesLagrange's equation $(1+h_v^2)h_{uu} - 2h_u h_v h_{uv} + (1+h_u^2)h_{vy} = 0.$ In the microscopic observations of our investigated plants, we determined that the druse crystals have a mathematically minimal surface.The minimal surface of these crystals are 'great stellated dodecahedron minimal surface' that is a mathematics definition. It is important example of minimal surfaces. These geometric shapes provide them with some important advantages such as taking up less space and durability.

Keywords: Stellated dodecahedron,microscopic structures,minimal surface, druse crystal.

Crystals are the storage of many mineral acid salts (inorganic salts) in many plants, such as chloride, phosphate, carbonate, silicate anhydrides and sulfates, which are formed as a result of metabolism. Calcium oxalates are the most well known among inorganic salts. Oxalic acid produced by metabolism is harmful to the plant. This acid combines with calcium from of the plant and turns into a crystal of calcium oxalate and becomes harmless to cells.All taxonomic levels of photosynthetic organisms from algae to angiosperms and developed gymnosperms contain Calcium oxalate (CaOx) crystals. These crystals deposited by the organisms are important to them.Microscopic investigations have shown that this biomineralization process seen in living things is not a simple physical-chemical precipitation of synthesized oxalite acid and Ca, which is of environmental origin. Crystals are formed in specific shapes and sizes by this biomineralization process (Vincent and Paul 2005, Bouropoulos *et al.* 2001).

Shaping of plant crystals are not a simple structure, they take different shapes. They are identifiable formations with special geometric shapes expressed in formulas in mathematics. Some of these is called as druse crystals. In our microscopic investigations, we observed that the druse crystals display a minimal surface feature, which has an important place in the field of mathematics.The term minimal surface is a mathematical concept and a surface that minimizes its occupied space. This means having zero average curvature. These surfaces appear as surfaces that minimize the total surface area with some restrictions (Lagrange 1760, Weisstein 2017 Fomenko and Tuzhilin 1991).

We observed in our research that the druse crystals have a stellated dodecahedron minimal surface which are the important examples of the minimal surfaces.These geometric shapes give them some advantages, such as taking up less space.

The geometric shapes that show minimal surface feature can be defined in several equivalent ways in R^3. The equivalent feature of minimal surface theory forms the intersection point of various mathematical disciplines such as differential geometry,

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calculus of variations, potential theory, complex analysis and mathematical physics (William and Joaquín 2011). The minimal surface that can be described as a surface with zero mean curvature parametrized as x= (u,v,h (u,v)) therefore satisfies Lagrange's equation, as x= (*u,v,h* (*u,v*))so satisfies Lagrange's equation,

$$(1 + h_v^2)h_{uu} - 2h_u h_v h_{uv} + (1 + h_u^2)h_{vy} = 0 \quad (1)$$

(Gray 1997).

Present study, we have examined the druse crystals that can only be seen with a microscope in some plant samples for them geometric structure. We have shown that the druse crystals, which are microscopic structures of plants, can be defined mathematically as geometric minimal surfaces.There are some mathematical studies in the literature about the visible external structures of plants (Oppenheimer 1986, Kaitaniemia 2000).

MATERIALS AND METHODS

Micromorphological structures of the plant samples(*Limonium sinuatum* (L.) Mill., *Gypsophila lepidioides* Boiss., *Arenaria sipylea* Boiss.) have investigated with light microscopy (LM).While determining the micromorphological structures of the plant, fresh and alcohol-preserved samples of the plant have been used for light microscopy studies.In the study, root and stem cells of the plant have been used for druse crystal.The anatomical sections stained with safranin-fast green. Then their photos have been taken using a motorized Leica DM 3000 microscope(Bozdag *et al.* 2016). In the study, literature informations have been used for photographs scanned under the electron microscope (Vincent and Paul 2005).

RESULTS AND DISCUSSION

In microscopic anatomy studies of our plant samples used in this study, we determined that some of their crystal have a mathematical minimal surfaces.The mathematical definitions of these geometric surfaces that we observed in the plants investigated are as follows. In addition, the geometric shapes obtained from these definitions are shown below also (Figure 1,2,3,4).

Minimal Surfaces:

Mathematical minimal surface can be also characterized as surface of minimal surface area for given boundary conditions.Meusnier found that a plane is a trivial minimal surface, and the first examples catenoid and helicoid in 1776 (Meusnier 1785). Hoffman demonstrated the existence of an infinite number of such surfaces by discovering a three-pronged type 1 minimally buried surface. Thus he showed that a surface can be parameterized using an isothermal parameterization. Before Hoffman, the catenoid, helicoid, and plane were minimal surfaces which are boundless and no self-intersections of known finite topology the only known (Hoffman,1987). Using an isothermal parameterization, a surface can be parameterized.If the coordinate functions $x_k$are harmonic i.e.,$\emptyset_k(\zeta)$ are analytic, such a parameterization is minimal (Hoffman and William, 1987). Therefore, a minimal surface can be represented by a triple of analytic functions such that

$$\emptyset_1^2 + \emptyset_2^2 + \emptyset_3^2 = 0. \quad (2)$$

The real parameterization is then obtained as

$$x_k = R \int \emptyset_k(\zeta)d\zeta. \quad (3)$$

On the other hand, for an analytic function f and a meromorphic function, the triple of functions

$$\emptyset_1(\zeta) = f(1 - g^2)\,(4) \quad (4)$$

$$\emptyset_2(\zeta) = if(1 + g^2) \quad (5)$$

$$\emptyset_3(\zeta) = 2fg \quad (6)$$

are analytic as long as f has a zero of order ≥m at every pole of g of order m. This result gives us, a minimal surface in terms of the Enneper-Weierstrass parameterization

$$R \int \begin{vmatrix} f(1-g^2) \\ if(1+g^2) \\ 2fg \end{vmatrix} d\zeta.$$

In microscopic anatomy studies of our plant samples we observed that their crystals with minimal surface have a stellated dodecahedron minimal surface (Figure 1,2,3,4).

Stellated dodecahedron minimal surface

A stellated dodecahedron was formally expressed in 1744 by the mathematician Leonhard Euler (Euler, 1774). A stellated dodecahedron is a minimal surface, that is, it occupies the least area when bounded by a closed space Gullberg (Jan, 1997). A stellated dodecahedron is basically formed by stellation from dodecahedron geometric structure.In three dimensions, stellation means building a new polyhedron from the existing one by the process of expanding elements such as sides or faces, usually symmetrically, until they are combined to form a new polygon or polyhedron.The new shape formed is a sign of the original. Kepler stellated the dodecahedron to obtain two of the regular star polyhedra (or Kepler-Poinsot polyhedra), who made this definition in 1619. Also Kepler stellated the octahedron obtaining the stella octangula or stellated octahedron (Kepler 1619).The most common dodecahedron are first stellation of the rhombic dodecahedron, small stellated dodecahedron and great stellated dodecahedron.We observed that the druse crystals of the plant specimens we investigated have usually a great stellated dodecahedron type minimal surface (Figure2,3).

The great stellated dodecahedron minimal surface

This the type of minimal surface is one of the Kepler-Poinsot solids. Also the great stellated dodecahedron isuniform polyhedron U_{52}, Wenninger model W_{41} and is the third dodecahedron stellation (Wenninger 1989). Its dual is the great icosahedron.

The great stellated dodecahedron has Schläfli symbol $\left\{\frac{5}{2},\ 3\right\}$ and Wythoff symbol $3\,|\,2\ \frac{5}{2}$.

It has 12 pentagrammic faces.Kepler rediscovered the great stellated dodecahedron (and published in his work Harmonice Mundi in 1619), then it was discovered again by Poinsot in 1810 (Kepler, 1619; Poinsot, 1810).

The great stellated dodecahedron can be constructed from one unit of dodecahedron by choosing 144 sets of five coplanar vertices, then discarding the sets whose edges correspond to the edges of the original dodecahedron. This gives us a 12 pentagrams side length

$$\emptyset^2 = (3+\sqrt{5})2^{-1},$$

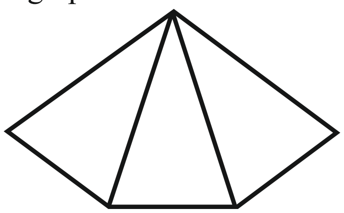
Where Ø is the golden ratio. Rescaling to give the pentagrams unit edge lengths, the circumradius of the great stellated dodecahedron is

$$R = \frac{1}{2}\sqrt{3}\emptyset^{-1} \qquad (8)$$

$$= \frac{1}{4}\sqrt{3}(\sqrt{5}-1), \qquad (9)$$

Where Ø is thegolden ratio.

As shown below, the skeleton of the great stellated dodecahedron is isomorphic to the dodecahedral graph.



Another way to build a great stellated

dodecahedron by magnification is to make 20 triangular pyramids with side length $\emptyset = (1+\sqrt{5})2^{-1}$

(the golden ratio) times the base, as illustrated above, and attach them to the sides of an icosahedron. The height of these pyramids is then $\sqrt{\frac{1}{6}(7+3\sqrt{5})}$.

The way to produce a solid with side lengths is to accumulate a unit of dodecahedron to construct a great stellated dodecahedron

$$S_1 = 1 \quad (10)$$

$$S_2 = \frac{1}{2}(1+\sqrt{5}). \quad (11)$$

The surface area and volume of such a great stellated dodecahedron are

$$S = 15\sqrt{5+2\sqrt{5}} \quad (12)$$

$$V = \frac{5}{4}(3+\sqrt{5}). \quad (13)$$

The dual of the great stellated dodecahedron (i.e., the great icosahedron) is one of the icosahedron stellations. Because the convex hull of the great stellated dodecahedron is a regular dodecahedron and the dual of the dodecahedron is the icosahedron(Wenninger 1983).

With this study we conducted on the micromorphological structures of some plant crystals, we observed that the microscopic structures of druse crystals have a minimal surface. The minimal surfaces that have an

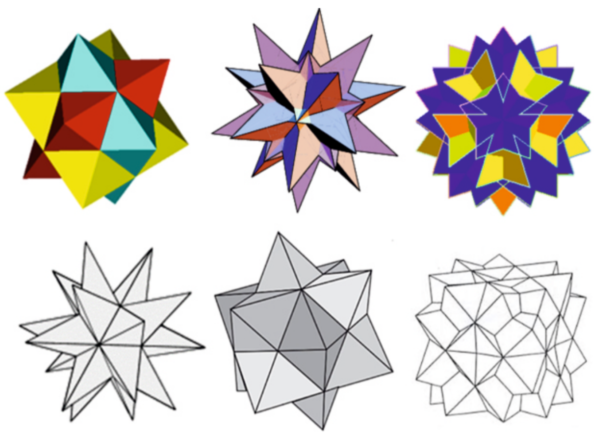


Figure 1. Geometric representations of stellated dodecahedron minimal surface(Wells, 1991)

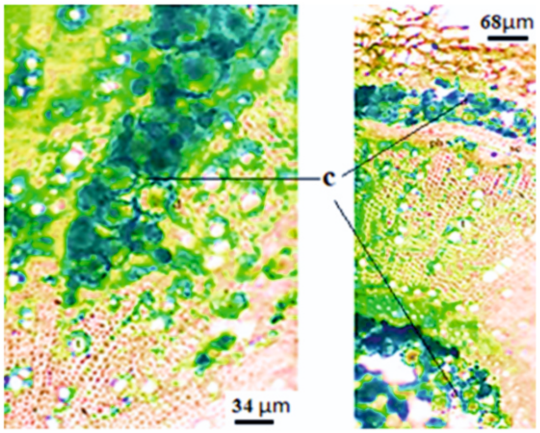


Figure 2. The druse crystals **(c)** (with LM) The cross-sections of the root of *Gypsophila lepidioides* Boiss.

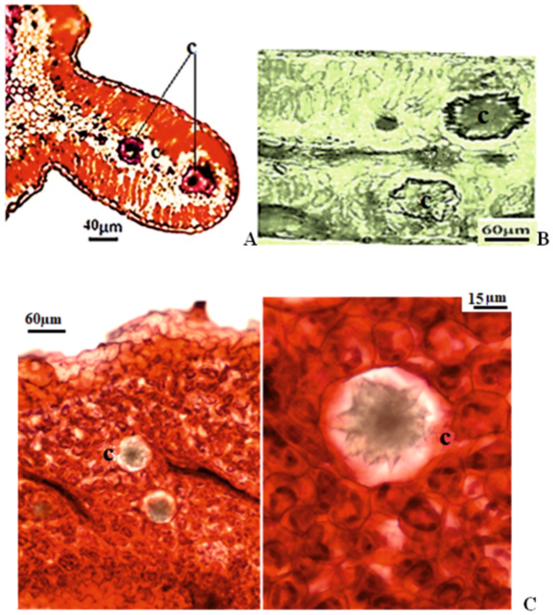


Figure 3. The druse crystals (**c**) (with LM) **A)**The cross-sections of the stem of *Limonium sinuatum* (L.) Mill **B)**The cross-section of the leaf of *Limonium sinuatum* (L.) Mill **C)**The cross-section of the leaf of *Arenaria sipylea* Boiss

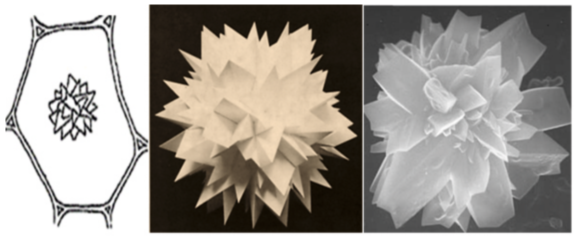


Figure 4. The druse crystals **A)**Illustration of micrographs of druse crystals in plant cell. **B)** Photograph of druse crystal model from Pawley (1975) **C)** *Peperomia* sp. (druse crystal with SEM) from Vincent and Paul (2005)

important place in mathematics are the smallest area "surface that we can fill a defined space given to us.These geometric formations are the strongest formations that not only minimize their area but also physically maintain durability. Minimal surfaces are defined by parametric equations.

In this study, it was seen that microscopic structures of the druse crystals that are the formations where some unused inorganic materials are stored can be defined as mathematically minimal surfaces.The druses crystals have often single but also multiple per cell, multifaceted conglomerate.These properties give them geometric surface features.

As a result of this study, we investigated druse crystals in some plant samples in terms of their geometric structures with a mathematical perspective. Finally, we determined that druse crystals have different stellated dodecahedron minimal surfaces which are the important examples of the minimal surfaces. These geometric shapes give them some important advantages, such as taking up less spacebut also allows them to be durable. Druse crystals in plants are where many mineral acid salts stored such as calcium oxalates. Thus, these structures with a minimal surface occupy little space and store a large amount of inorganic materials that occur as a result of metabolism andunused inorganic substances.

As a result of our literature review on our subject of study, we have come across studies on the use of minimal surfaces in today's architecture and similarly in different areas. These works are based on morphological features considering the advantages of the minimal surfaces that are the subject of geometry. However, these studies are not micromorphological studies, but the investigations of minimal surface properties of objects that can be seen with the naked eye (Pires *et al.,* 2017;Melo and Andrade, 2018).There are very few mathematical studies on the micromorphological structures of plants in the literature (Korn and Spalding, 1973; Özdemir, 2018). However, there is no study on the presence of minimal surfaces in the druse chyrstal microscopic structures that are invisible to the naked eye, which is our subject of study, and which we observe with a microscope.

Thanks to this study, minimal surfaces which is a geometric concept, have been extracted from the theoretical form expressed by mathematical formulas and they have been shown with concrete examples in nature. At the same time, again with this study, a different perspective has been brought by evaluating the micromorphological structures of plants as mathematically by expressing it in mathematical formulas.As a result, other researchers who will work on the subject related to this issue will find a different comparison chance with our the search .

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